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# Multiparticle production process in high energy nucleus-nucleus collisions

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**Abstract** : We apply here an impact-parameter analysis depending on the parton two-Fireball model. In this model, each of the colliding hadrons is considered as a bundle of point-like particles (partons). Only those partons in the overlapping volume from the colliding hadrons participate in the interaction, which are assumed to be stopped in CMS. Therefore, two excited intermediate states (fireballs) are produced which later on decay to produce the observed created secondaries The parameters characterizing the multiparticle production process for $Li^7$, $C^{12}$ and $O^{16}$ in nuclear emulsion have been estimated and compared with the experimental data

**Keywords** : Nucleus-nucleus collisions, multiparticle production process, impact parameter analysis

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## 1. Introduction

Ii is well established that nucleons are composite objects consisting of a fixed number of partons [1]. This nucleon structure have been used in different models [2,3] along with other assumptions to describe hadron-hadron interactions. One of these models is the parton two fireball model (PTFM) proposed by Hagedorn [4,5]. PTFM along with the impact parameter analysis have been used in studying the high energy proton-proton and proton-nucleus interactions by Tantawy [6] and El-Bakry [7]. It has also been used to study high energy hadron-hadron and hadron-nucleus interactions by El-Mashad [8]. All these studies show good predictions of the measured parameters. In the present work, we extend this model to study the multiparticle production process in nucleus-nucleus high energy interactions.

## 2. The model

We apply here an impact-parameter analysis depending on the parton two fireball model to study nucleus-nucleus interactions at high energies. The basic assumptions in this model can be summarized as follows :

(i) The colliding hadrons are composed of a fixed number of point like particles called partons. These partons can be treated as losely bound states. At high energies, partons have negligible transverse momenta [1].

(ii) Only those partons within the overlapping volume of the two interacting hadrons, have the probability to interact which are assumed to be stopped in the CMS. Therefore, their CM-kinetic energy will be consumed in the excitation of the produced two fireballs.

(iii) Each fireball will decay into a number of newly created particles (mainly poins) with an isotropic angular distribution in its own rest frame.

It is now clear that in this model, the mass of the produced fireballs and consequently the number of the created particles are functions of the overlapping volume at certain incident energy. The overlapping volume is defined by the incident impact-parameter. Then using the above assumptions, we can investigate the multiparticle production process in nucleus-nucleus interactions.

### 2.1. Impact-parameter distribution :

Let us assume that the interacting nuclei (projectile and target) at rest are spheres of radii $R_1$ and $R_2$ respectively. Then the statistical probability of impact parameter ($b$) within an interval $db$ is given by

$$P(b)db = 2bdb/\left(R_1 + R_2\right)^2,$$

i.e.
$$P(b)db = 2bdb \Big/ \left[r_0^2\left(A_1^{1/3} + A_2^{1/3}\right)^2\right], \tag{1}$$

where $r_0 = (1.22 \rightarrow 1.5)$ fm and $A_1$ and $A_2$ are the mass numbers of the two interacting nuclei respectively. In terms of a dimensionless impact parameter ($x$) defined as $X = \frac{b}{r_0}$, eq. (1) becomes

$$P(x)dx = 2xdx \Big/ \left(A_1^{1/3} + A_2^{1/3}\right)^2. \tag{2}$$

If one assumes that the partons from the incident nucleus in the overlapping volume will interact with the nuclear matter of the target, then we can calculate the overlapping volume $v(x)$ in the incident nucleus rest frame. Then, we can calculate the fraction of partons participating in the interaction ($z$) as a function of ($x$), as

$$Z(x) = \frac{v(x)}{v_0} = \left(\frac{1}{2}A_1 + \frac{3}{4}A_1^{2/3}A_2^{1/3} - \frac{1}{4}A_2\right)$$

$$+ \left(\frac{3}{4}A_2^{2/3} - \frac{3}{4}A_1^{2/3}\right)x - \frac{3}{4}A_2^{1/3}x^2 + \frac{1}{4}x^3, \quad (3)$$

where $v_0$ is the volume of the nucleon.

From eqs. (2) and (3) we can get the $z$-function distribution as

$$P(z)dz = \frac{2xdz}{\left(A_1^{1/3} + A_2^{1/3}\right)^2 \left[\left(\frac{3}{4}A_2^{2/3} - \frac{3}{4}A_1^{2/3}\right) - \frac{3}{2}A_2^{1/3}x + \frac{3}{4}x^2\right]} \quad (4)$$

We have calculated eq. (4) for $Li^7$, $C^{12}$ and $O^{16}$ on nuclear emulsion.

**Table 1.** The values of the coefficients $C_k$ in eq. (5)

| Type of interaction | $C_{-1}$ | $C_0$ | $C_1$ | $C_2$ | $C_3$ |
|---|---|---|---|---|---|
| $Li^7$-Em | 0.69 | –0 41 | 0.17 | –0.03 | 0.0021 |
| $Li^7$-CNO | 0.135 | 0.022 | 0 035 | –0.01 | 0.0009 |
| $Li^7$-AgBr | 0 143 | 0.039 | 0 0044 | –0 0027 | 0 0004 |
| $Li^7$-C | 0.112 | 0 071 | 0.0105 | –0.0057 | 0.00058 |
| $Li^7$-N | 0.117 | 0.055 | 0 02 | –0.0081 | 0.0008 |
| $Li^7$-O | 0.105 | 0.075 | 0.011 | –0.0067 | 0.00075 |
| $Li^7$-Ag | 0 255 | –0 232 | 0 169 | –0 039 | 0 0031 |
| $Li^7$-Br | 0.309 | –0.281 | 0.185 | –0.0418 | 0 0033 |
| $C^{12}$-Em | 0.058 | 0 047 | 0 0047 | –0.0016 | 0.00012 |
| $C^{12}$-CNO | 0 183 | 0.013 | 0.0068 | –0.001 | 0 00004 |
| $C^{12}$-AgBr | 0.063 | 0.048 | –0.0004 | –0.0004 | –0 00004 |
| $C^{12}$-C | 0.068 | 0 105 | –0 019 | 0.0019 | –0.00007 |
| $C^{12}$-N | 0.073 | 0.094 | –0.014 | 0.0012 | –0.000037 |
| $C^{12}$-O | 0.07 | 0 09 | –0.012 | 0.00078 | –0.0000116 |
| $C^{12}$-Ag | 0 121 | 0.0023 | 0 019 | –0 0037 | 0 00021 |
| $C^{12}$-Br | 0.147 | –0.028 | 0.029 | –0.0049 | 0.00027 |
| $O^{16}$-Em | 0.068 | 0 05 | –0.003 | –0.000014 | 0 00001 |
| $O^{16}$-CNO | 0.083 | 0.066 | –0.007 | 0.00044 | –0.00001 |
| $O^{16}$-AgBr | 0.24 | –0.026 | 0.008 | –0.0006 | 0.00002 |
| $O^{16}$-C | 0.059 | 0 095 | –0.016 | 0.0014 | –0.00005 |
| $O^{16}$-N | 0.066 | 0.085 | –0.013 | 0.001 | –0 00003 |
| $O^{16}$-O | 0.07 | 0.077 | –0.0099 | 0.0007 | –0.00002 |
| $O^{16}$-Ag | 0.104 | 0.017 | 0.0077 | –0.0013 | 0.00006 |
| $O^{16}$-Br | 0.125 | –0.0018 | 0.0123 | –0.0018 | 0.000076 |

Since eq. (4) is not a simple function of $z$, to get analytic equation for the $z$-function distribution, we used the fitting procedure to the curves drawn from eq. (4) for all collisions which yields

$$P(z)dz = \sum_{k=-1}^{3} C_k z^k dz. \tag{5}$$

The values of the coefficients $C_k$ are given in Table 1.

*2.2. Shower particle production in N-N collisions :*

After the collision takes place, the partons within the overlapping volume stop in the CMS and their K.E changes an excitation energy to produce two intermediate states (fireballs).

The produced fireballs will radiate the excitation energy into a number of newly created particles which are mainly pions. We assume that each fireball will decay in its own rest frame into a number of pions with an isotropic angular distribution plus one baryon. The number of created pions will be defined by the fireball rest mass ($M_f$) and the mean energy consumed in the creation of each pion ($\varepsilon$).

The excitation energy from each fireball is

$$M_f - m = T_0\, z(x), \tag{6}$$

where $T_0$ is the kinetic energy and $m$ is the proton mass at rest.

The number of pions from each fireball ($n_0$) will be given by

$$n_0(z) = \frac{T_0\, Z(x)}{\varepsilon} = \frac{Z(x)Q}{2\varepsilon}, \tag{7}$$

where $Q$ is the total K.E in CMS ($= 2T_0$), since all the experimental measurements are concerned with the charged (shower) particles in the final state. Therefore, we have to assume some distribution (*e.g.* Binomial and Poisson distribution) for the shower particles ($n_s$) in the final state of the interaction at any impact parameter, out of total created particles ($n_0$).

Accordingly, we shall investigate the probability of getting shower particles ($n_s$) from the two fireballs as follows :

From eqs. (5) and (7) we get,

$$P(n_0) = \sum_{k=0}^{3} \left(\frac{2\varepsilon}{Q}\right)^{k+1} c_k \frac{(n_0+1)^{k+1} - (n_0)^{k+1}}{(k+1)} + C_{-1} \ln\left(\frac{n_0+1}{n_0}\right). \tag{8}$$

Let us assume different probability distributions for creation of shower charged pions from one fireball $\psi(n_2)$, such as, (a) binomial distribution of the form :

$$\Psi_{(n_2)} = \frac{N!}{n_2!(N-n_2)!} Pn_2 \, q^{(N-n_2)}, \tag{9}$$

where $N$ is the number of created particles from one fireball = $n_0/2$,

$n_2$ is the number of pairs of charged particles,

$P$ and $q$ are the probabilities that the pair of particles is charged or neutral, respectively.

or (b) Poisson distribution of the form :

$$\Psi_{(n_2)} = \frac{N^{n_2}}{n_2!} P^{n_2} e^{-NP}. \tag{10}$$

Now, the number of charged particles from one fireball will be given by

$$n = 2n_2 + 1.$$

Then the distribution of shower particles from one fireball will be

$$\Phi(n) = \sum_{n_0} \Psi(n_2) P(n_0). \tag{11}$$

Because of charge conservation, $\Phi(n)$ at $n_0$ = even, is equal to $\Phi(n)$ at $(n_0 + 1)$. Therefore, we can calculate the probability of getting any number of shower particles $(n_s)$ from the two fireballs as

$$P(n_s) = \sum_{n=1}^{n_s} \Phi(n)\Phi(n_s - n). \tag{12}$$

The above equations can be used for studying all the characteristics of the shower particle production process such as multiplicity distribution, average multiplicity, KNO-scaling as well as the multiplicity dispersion.

*2.2.1. The shower particle multiplicity distribution* :

If we assume that the energy required for creation of one pion in the fireball rest frame $(\varepsilon)$ increases with the multiplicity size $(n_0)$ as

$$\varepsilon = an_0 + b, \tag{13}$$

where $a$ and $b$ are free parameters which can be evaluated to give the best fitting with the experimental data, *e.g.* $a$ = 0.04 and $b$ = 0.35 gives good fitting for hadron-hadron and hadron-nucleus interactions [8].

We have calculated the shower multiplicity distribution (eq. 12) for $C^{12}$ incident on target emulsion $(\langle A \rangle = 70)$ at $P_L$ = 4.5 A Gev/c. The results of these calculations have been shown in Figure 1 compared with the corresponding experimental data [9].

Figure 1 shows a qualitative agreement of predicted distributions (using binomial distribution eq. (9) and poisson distribution eq. (10)) with the measured ones. There is some

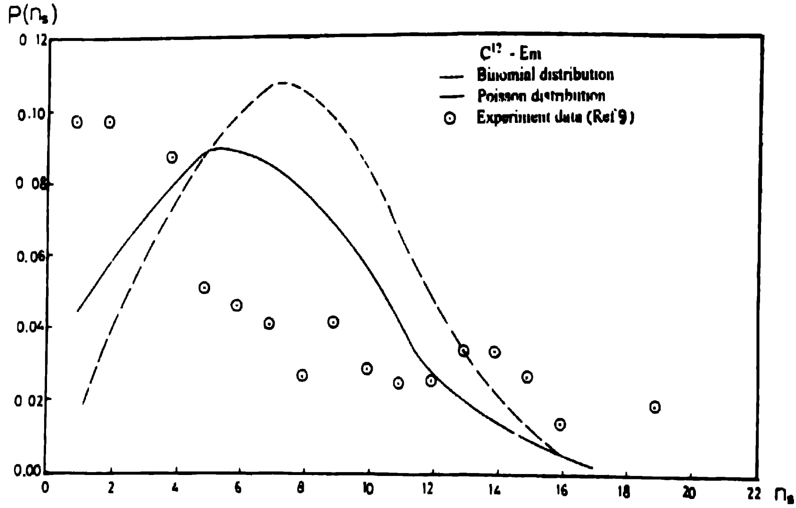


**Figure 1.** $n_s$-distribution for $C^{12}$-Em at $P_L = 4.5$ A Gev/c.

deviation of the numerical values between the calculated and measured distributions. We refer this disagreement to the unspecification of the target. Thus, we can recalculate the shower particle multiplicity distribution for the emulsion groups CNO and AgBr. The results of these calculations for $Li^7$, $C^{12}$ and $O^{16}$ in emulsion at 4.5 A Gev/c, are shown in Figures 2(a–c) together with the corresponding experimental data [9,11–13].

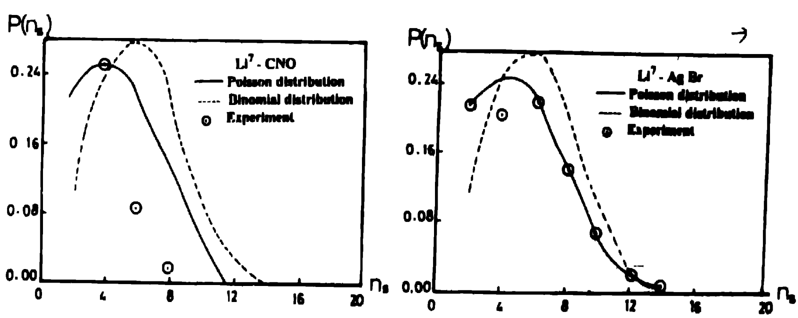


**Figure 2(a).** $n_s$-distribution for $Li^7$ with emulsion groups (CNO, AgBr) at $P_L = 4.5$ A Gev/c.

For further refinement of the model predictions, we have calculated $n_s$-distribution from the emulsion components percentage as follows :

(i) For a specific projectile, the $z$-function distribution can be calculated for this projectile with the components of the target emulsion separately *i.e.* (C–N–O–Ag–Br).

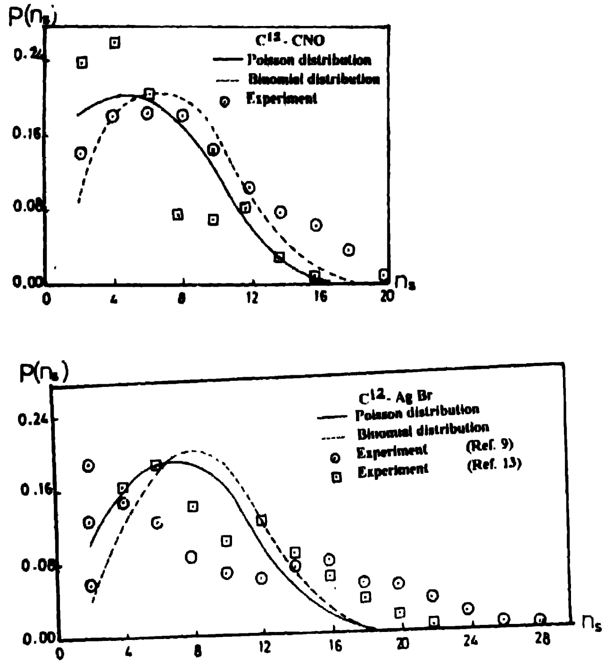


**Figure 2(b).** $n_s$-distribution for $C^{12}$ with emulsion groups (CNO, AgBr) at $P_L = 4.5$ A Gev/c.

(ii) Using the same scheme, we can calculate the shower particle multiplicity distribution for each projectile ($Li^7$, $C^{12}$, $O^{16}$) with the emulsion components.

(iii) From the emulsion components percentage [10], we can combine these distributions to get the final shower particle distribution for this projectile with target emulsion. The results of these calculations for $Li^7$, $C^{12}$ and $O^{16}$ in emulsion at $P_L$ = 4.5 A Gev/c using eq. (10), are represented in Figure 3 which shows good agreement with the corresponding experimental data [9,11–13].

In Figure 3, we compare our results for shower particle multiplicity distribution in $Li^7$-Em collisions with those obtained by the nucleon-nucleus superposition method (14). In this method, the multiplicity distribution is given by

$$P_{A_p A_T}(n_s) = \sum_{N=1}^{n_p} P_p(N) P(N, n_s), \tag{14}$$

where $P_P(N)$ is the probability for the interaction of $N$ out of $A_P$ projectile nucleons, given by

$$P_p(N) = \sigma(N, A_p)/\sigma_{A_p A_T}, \tag{15}$$

and $A_P$, $A_T$ are the mass number of the projectile and target respectively.

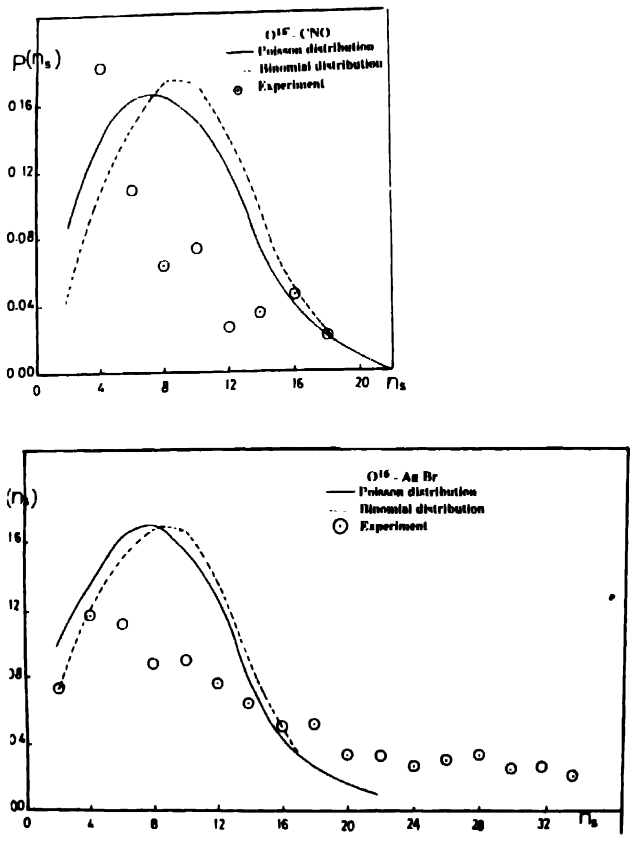


**Figure 2(c).** $n_s$-distribution for $O^{16}$ with emulsion groups (CNO, AgBr) at $P_L = 4.5$ A Gev/c

*2.2.2. Average shower particles multiplicity ($< n_s >$) and multiplicity dispersion ($D$) :*

Using the shower particles multiplicity distribution described above with the Poisson distribution of emission, we have calculated the average shower particles multiplicity through relation

$$\langle n_s \rangle = \sum n_s P_{(n_s)}. \tag{16}$$

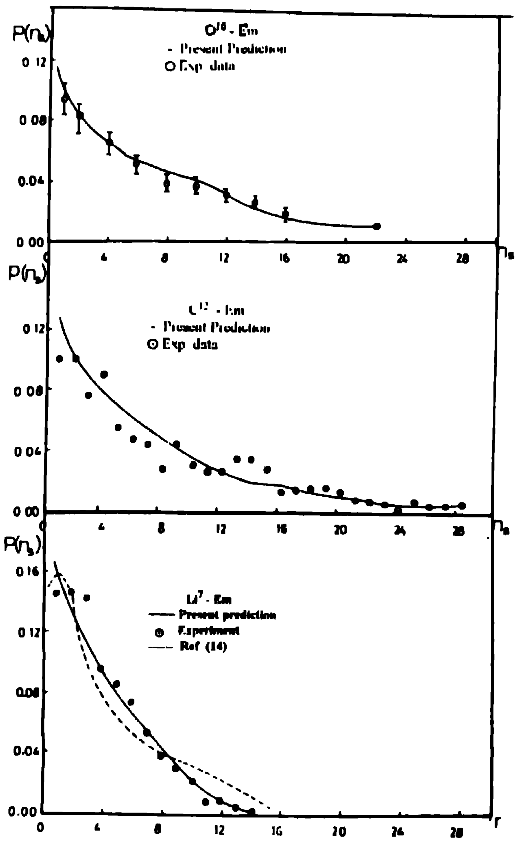


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**Figure 3.** $n_s$-distribution for $Li^7$, $C^{12}$ and $O^{16}$ with emulsion (considering emulsion components percentage) at $P_L$ = 4.5 A Gev/c.

Table 2 shows the calculated $<n_s>$ for the considered interactions together with the corresponding measured values for comparison.

**Table 2.** The calculated and the measured values of average shower multiplicity and dispersion parameter.

| Type of interaction | $< n_s >$ th | $< n_s >$ exp | $D_{th}$ | $D_{exp}$ |
|---|---|---|---|---|
| $Li^7$-CNO | 4.88 | 2.16 ± 0.13 | 2.97 | |
| $Li^7$-AgBr | 5.61 | 4.63 ± 0.195 | 2.83 | |
| $Li^7$-Em | 3.88 | 3.6 ± 0.11 | 3.32 | 3.07 ± 0.12 |
| $C^{12}$-CNO | 6.41 | 5.04 ± 0.21 | 3.4 | 3.66 ± 0.15 |
| $C^{12}$-AgBr | 7.76 | 8.92 ± 0.25 | 3.5 | 5.17 ± 0.18 |
| $C^{12}$-Em | 7.01 | 7.67 ± 0.13 | 6.41 | 7.10 ± 0.23 |
| $O^{16}$-CNO | 8.47 | 5.99 ± 0.41 | 4.28 | 6.16 ± 0. |
| $O^{16}$-AgBr | 8.62 | 12.87 ± 0.63 | 4.33 | 10.01 ± |
| $O^{16}$-Em | 6.7 | 9.6 ± 0.4 | 5.73 | |

Included in this table are also the dispersion parameters defined as

$$D = \left[\langle n\rangle^2 - \langle n^2\rangle\right]^{1/2}. \tag{17}$$

Table 2 includes the calculated values of the dispersion $D$ due to our predictions together with the corresponding experimental data. From this table, we can conclude that

(i) The calculated values for $<n_s>$ and $D$ agree with the corresponding experimental ones only at specification of target (C–N–O–Ag–Br) while it is in qualitative agreement for unspecified target.

(ii) $<n_s>$ and $D$ increase as projectile and target mass numbers increase which reflects that $<n_s>$ is strongly dependent on each of beam and target mass numbers.

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